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# Some topological indices of edge-neighborhood corona of two graphs

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**Abstract**

In this paper, we introduce edge-neighborhood corona of two graphs and compute its Wiener index, degree distance index and Gutman index.

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**keywords**: Corona, Wiener index, Zagreb indices, PI index, degree distance index, Gutman index.

## 1 Introduction

Let $G$ be a connected simple graph with vertex set $V(G)$ and edge set $E(G)$. The distance $d_G(u,v)$ between two vertices $u$ and $v$ in $G$ is the number of edges in a shortest path between $u$ and $v$ in $G$. The degree $d_G(u)$ of a vertex $u$ in $G$ is the number of vertices that are adjacent to $u$ in $G$. A topological index is a numerical quantity related to a graph which is invariant under graph automorphisms. The oldest topological index is the Wiener index $W(G)$ introduced by Wiener [20] during 1947, by the name path number. Wiener used it to determine the boiling point of paraffin. It is a distance based graph invariant defined as the sum of distance between all pairs of vertices in $G$ i.e.,

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v).$$

The first and second Zagreb indices are the degree based topological indices introduced in 1972, by Gutman and Trinajstić [14] and these are defined as

$$M_1(G) = \sum_{v_i\in V(G)} d_G^2(v_i) = \sum_{e_i=v_lv_m\in E(G)} (d_G(v_l)+d_G(v_m))$$

and

$$M_2(G) = \sum_{e_i=v_lv_m\in E(G)} d_G(v_l)d_G(v_m), \text{ respectively.}$$

The degree distance index $DD(G)$ and Gutman index $Gut(G)$ of a graph are weighted versions of Wiener index, which are defined as follow:

$$DD(G) = \sum_{\{u,v\}\subseteq V(G)} (d_G(u)+d_G(v))d_G(u,v)$$

and

$$Gut(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u)\ d_G(v)\ d_G(u,v).$$

The degree distance index which is a degree distance based graph invariant, was introduced independently by Dobrynin, Kochetova [9] and Gutman [12]. The Gutman index, earlier known as Schultz index of the second kind was introduced in 1994 by Gutman [12]. More details about Wiener index and its variants can be found in [3, 6, 7, 8, 9, 10, 13, 17, 19] and the references cited therein. The vertex Padmakar-Ivan(PI) index [4] of a graph $G$ is defined as

$$PI(G) = \sum_{e_i=v_l v_m \in E(G)} [n_{e_i}(v_l|G) + n_{e_i}(v_m|G)],$$

where $n_{e=uv}(u|G)$ is the number of vertices in $G$ that are closer to $u$ than $v$ in $G$. The edge-Wiener index $W_e(G)$ [5] of a graph $G$, is the sum of all distances between unordered pairs of vertices in the line graph $L(G)$ of $G$. Equivalently, edge-Wiener index of $G$ is the Wiener index of the line graph $L(G)$ of $G$. i.e,

$$W_e(G) = W(L(G)) = \sum_{\{xy,uv\}\in E(G)} (min\{d_G(x,u), d_G(x,v), d_G(y,u), d_G(y,v)\} + 1).$$

In literature, many topological indices are introduced and are used as molecular descriptors.

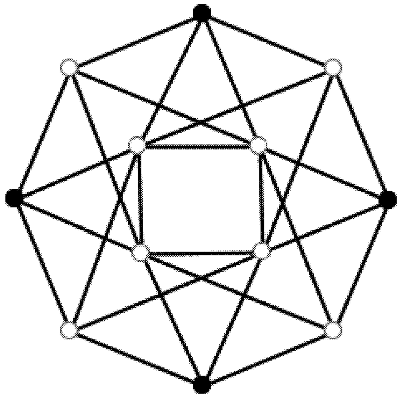
The corona [11] of two graphs $G_1$ and $G_2$ is the graph obtained by taking one copy of $G_1$, $|V(G_1)|$ copies of $G_2$ and joining each $i$-th vertex of $G_1$ to every vertex in the $i$-th copy of $G_2$. The edge corona $G_1 \diamond G_2$ [21] of two graphs $G_1$ and $G_2$, is the graph obtained by taking one copy of $G_1$ and $|E(G_1)|$ copies of $G_2$ and joining end vertices of $i$-th edge of $G_1$ to every vertex in the $i$-th copy of $G_2$. The neighborhood corona [15] of two graphs $G_1$ and $G_2$ denoted by $G_1 * G_2$, is a variant of corona of two graphs and is defined as the graph obtained by taking one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$, and joining every neighbor of the $i$-th vertex of $G_1$ to every vertex in the $i$-th copy of $G_2$. In literature, many variants of corona product as like edge corona and neighborhood corona are introduced. Recently, various graph invariants of corona product of two graphs and its variants have been studied, for example, see [1, 2, 16, 18, 22].

Motivated by this, in this paper, we introduce a new variant of corona product called as edge-neighborhood corona of two graphs, and compute the Wiener index, degree distance index and Gutman index of edge-neighborhood corona. Let $G_1$ be a graph with vertex set $V(G_1) = \{v_1, v_2, \ldots, v_{n_1}\}$ and edge set $E(G_1) = \{e_1, e_2, \ldots, e_{m_1}\}$. Let $G_2$ be a graph with vertex set $V(G_2) = \{u_1, u_2, \ldots, u_{n_2}\}$ and edge set $E(G_2) = \{e'_1, e'_2, \ldots, e'_{m_2}\}$. The edge-neighborhood corona of two graphs $G_1$ and $G_2$ denoted by $G_1 \circledast G_2$, is the graph constructed using $G_1$ and $G_2$ as follows:

(a) Take one copy of $G_1$. For each vertex $v_i$ of $G_1$ and also for each edge $e_j$ of $G_1$, take a copy of $G_2$, call them as the $i$-th vertex copy of $G_2$ and the $j$-th edge copy of $G_2$, respectively.

(b) Join every neighbor of a vertex $v_i$ of $G_1$ to every vertex in the $i$-th vertex copy of $G_2$, and also, join the end vertices of each edge $e_j$ of $G_1$ to every vertex in the $j$-th edge copy of $G_2$.

**Example 1.1.** *The edge-neighborhood corona $C_4 \circledast K_1$.*



$Fig.1 : Graph\ C_4 \circledast K_1.$

## 2 Preliminaries

To prove our main results we need the following definitions (cf. [1, 2]) and lemmas whose proofs follow directly from the definition of edge-neighborhood corona.

**Definition 2.1.** *For a graph $G$,*

$$E_\Delta(G) := \{e \in E(G) : e \text{ is contained in a triangle of } G\},$$
$$T_1(G) := \sum_{uv \in E_\Delta(G)} (d(u) + d(v)),$$
$$T_2(G) := \sum_{uv \in E_\Delta(G)} d(u)d(v)$$

*and*

$$C(G) := \sum_{\substack{w \in V(G) \\ e=uv \in E(G) \\ d_G(w,v)=d_G(u,w)}} d_G(w).$$

For $i = 1, 2, \ldots, m_1$ and $j = 1, 2, \ldots, n_1$, we denote the vertex set of the $i$-th edge copy of $G_2$ by $V_{i_e}(G_2) = \{u_{i1}, u_{i2}, \ldots, u_{in_2}\}$ and the vertex set of the $j$-th vertex copy of $G_2$ by $V_{j_v}(G_2) = \{w_{j1}, w_{j2}, \ldots, w_{jn_2}\}$, where $u_{ik} = w_{jk} = u_k$ for $k = 1, 2, \ldots, n_2$. The following lemmas gives the degree of vertex and distance between two vertices in $G_1 \circledast G_2$.

**Lemma 2.2.** *Let $G = G_1 \circledast G_2$, $i = 1, 2, \ldots, m_1$ and $j = 1, 2, \ldots, n_1$. Then*

*a.* $d_G(v_j) = (2n_2 + 1)d_{G_1}(v_j)$.

*b.* $d_G(u_{ik}) = d_{G_2}(u_k) + 2,\ \forall\ u_{ik} \in V_{i_e}(G_2)$.

*c.* $d_G(w_{jk}) = d_{G_2}(u_k) + d_{G_1}(v_j),\ \forall\ w_{jk} \in V_{j_v}(G_2)$.

**Lemma 2.3.** *Let $G = G_1 \circledast G_2$. Then* *a.* $d_G(v_i, v_j) = d_{G_1}(v_i, v_j), \; \forall \; v_i, v_j \in V(G_1)$.

*b.* *for* $i = 1, 2, \ldots, m_1, \; j = 1, 2, \ldots, n_1, \; d_G(u_{ik}, u_{im}) = \begin{cases} 1, & \text{if } u_k u_m \in E(G_2), \\ 2, & \text{if } u_k u_m \notin E(G_2). \end{cases}$ *Also,*

$$d_G(w_{jk}, w_{jm}) = \begin{cases} 1, & \text{if } u_k u_m \in E(G_2), \\ 2, & \text{if } u_k u_m \notin E(G_2). \end{cases}$$

*c.* *for* $u_{ik} \in V_{i_e}(G_2)$, *and* $u_{jm} \in V_{j_e}(G_2)$, $i, j = 1, 2, \ldots, m_1$ *and* $i \neq j$, $d_G(u_{ik}, u_{jm}) = d_{G_1}(e_i, e_j) + 2$.

*d.*
$$d_G(w_{ik}, w_{jm}) = \begin{cases} 3, & \text{if } v_i v_j \in E(G_1) \text{ and } v_i v_j \notin E_\Delta(G_1), \\ 2, & \text{if } v_i v_j \in E_\Delta(G_1), \\ d_{G_1}(v_i, v_j), & \text{if } v_i v_j \notin E(G_1), \end{cases}$$
*for* $i, j = 1, 2, \ldots, n_1, \; i \neq j$ *and* $k, m = 1, 2, \ldots, n_2$.

*e.* *for* $i = 1, 2, \ldots, m_1, e_i = v_l v_m, u_{ij} \in V_{i_e}(G_2)$ *and* $v_k \in V(G_1)$,
$$d_G(u_{ij}, v_k) = \begin{cases} (1/2)(d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1), & if \;\; d(v_l, v_k) \neq d(v_k, v_m), \\ (1/2)(d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m)) + 1, & if \;\; d(v_l, v_k) = d(v_k, v_m). \end{cases}$$

*f.*
$$d_G(w_{ij}, v_k) = \begin{cases} d_{G_1}(v_i, v_k), & \text{if } v_i \neq v_k, \\ 2, & \text{if } v_i = v_k. \end{cases}$$
$\forall \; w_{ij} \in V_{i_v}(G_2), i = 1, 2, \ldots, n_1$ *and* $v_k \in V(G_1)$.

*g.* *for* $i = 1, 2, \ldots, m_1, e_i = v_l v_m, u_{ij} \in V_{i_e}(G_2), k = 1, 2, \ldots, n_1$ *and* $w_{km} \in V_{k_v}(G_2)$,
$$d_G(u_{ij}, w_{km}) = \begin{cases} (1/2)(d_{G_1}(v_k, v_l) + d_{G_1}(v_k, v_m) + 1), & \text{if } d(v_k, v_l) \neq d(v_k, v_m) \\ & \text{and } v_k \neq v_l \text{ or } v_m, \\ (1/2)(d_{G_1}(v_k, v_l) + d_{G_1}(v_k, v_m)) + 1, & \text{if } d(v_k, v_l) = d(v_k, v_m) \\ & \text{and } v_k \neq v_l \text{ or } v_m, \\ 2, \; if \;\; v_k = v_l \text{ or } v_m. & \end{cases}$$

## 3 Main Results

Let $G = G_1 \circledast G_2$. In this section, we compute Wiener index, degree distance index and Gutman index of edge-neighborhood corona of two graphs.

**Theorem 3.1.** *The Wiener index of* $G_1 \circledast G_2$ *is given by*

$$W(G) = (n_2+1)^2 W(G_1) + n_2^2(W_e(G_1) - |E_\Delta(G_1)|) + \frac{n_2}{2}\Big\{(n_2+1)DD(G_1) - (n_2+1)PI(G_1) + n_2(m_1^2 + (2n_1+9)m_1 + 2n_1) + 2((n_1-1)m_1 + n_1)\Big\} - m_2(n_1+m_1).$$

*Proof.* From Lemma 2.3, we have

$$W_1 := \sum_{\{v_i, v_j\} \subseteq V(G_1)} d_G(v_i, v_j) = \sum_{\{v_i, v_j\} \subseteq V(G_1)} d_{G_1}(v_i, v_j) = W(G_1),$$

$$
\begin{aligned}
W_2 &:= \sum_{e_i \in E(G_1)} \sum_{\{u_{ij}, u_{ik}\} \subseteq V_{i_e}(G_2)} d_G(u_{ij}, u_{ik}) \\
&= \sum_{e_i \in E(G_1)} \Big\{ 2 \sum_{\{u_j, u_k\} \subseteq V(G_2)} - \sum_{u_j u_k \in E(G_2)} \Big\} \\
&= m_1(n_2(n_2 - 1) - m_2),
\end{aligned}
$$

$$
W_3 := \sum_{v_i \in V(G_1)} \sum_{\{w_{ij}, w_{ik}\} \subseteq V_{i_v}(G_2)} d_G(w_{ij}, w_{ik}) = n_1(n_2(n_2 - 1) - m_2) \quad \text{(similar to } W_2\text{)},
$$

$$
\begin{aligned}
W_4 &:= \sum_{\{e_i, e_k\} \subseteq E(G_1)} \sum_{\substack{u_{ij} \in V_{i_e}(G_2) \\ u_{km} \in V_{k_e}(G_2)}} d_G(u_{ij}, u_{km}) \\
&= \sum_{\{e_i, e_k\} \subseteq E(G_1)} \sum_{\substack{u_{ij} \in V_{i_e}(G_2) \\ u_{km} \in V_{k_e}(G_2)}} (d_{G_1}(e_i, e_k) + 2) \\
&= n_2^2 \sum_{\{e_i, e_k\} \subseteq E(G_1)} (d_{G_1}(e_i, e_k) + 2) \\
&= n_2^2 (W_e(G_1) + m_1(m_1 - 1)/2),
\end{aligned}
$$

$$
\begin{aligned}
W_5 &:= \sum_{\{v_i, v_k\} \subseteq V(G_1)} \sum_{\substack{w_{ij} \in V_{i_v}(G_2) \\ w_{km} \in V_{k_v}(G_2)}} d_G(w_{ij}, w_{km}) \\
&= n_2^2 \Big\{ \sum_{\{v_i, v_j\} \subseteq V(G_1)} d_{G_1}(v_i, v_k) + \sum_{v_i v_j \in E(G_1)} 2 - \sum_{v_i v_j \in E_\Delta(G_1)} 1 \Big\} \\
&= n_2^2 (W(G_1) + 2m_1 - |E_\Delta(G_1)|),
\end{aligned}
$$

$$
\begin{aligned}
W_6 &:= \sum_{e_i = v_l v_m \in E(G_1)} \sum_{\substack{u_{ij} \in V_{i_e}(G_2) \\ v_k \in V(G_1)}} d_G(u_{ij}, v_k) \\
&= (n_2/2) \sum_{e_i \in E(G_1)} \Big\{ \sum_{\substack{v_k \in V(G_1) \\ d(v_l, v_k) \neq d(v_k, v_m)}} (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1) + \\
&\qquad \sum_{\substack{v_k \in V(G_1) \\ d(v_l, v_k) = d(v_k, v_m)}} \Big( d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 2 \Big) \Big\} \\
&= (n_2/2) \sum_{e_i \in E(G_1)} \Big\{ \sum_{v_k \in V(G_1)} (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m)) + 2n_1 - \\
&\qquad (n_{e_i}(v_l | G_1) + n_{e_i}(v_m | G_1)) \Big\} \\
&= \frac{n_2}{2} \Big\{ \sum_{e_i \in E(G_1)} \sum_{v_k \in V(G_1)} (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m)) + 2n_1 m_1 - PI(G_1) \Big\} \\
&= \frac{n_2}{2} \Big\{ \sum_{v_i \in V(G_1)} \sum_{v_i v_j \in E(G_1)} \sum_{v_k \in V(G_1)} d_{G_1}(v_i, v_k) + 2n_1 m_1 - PI(G_1) \Big\}
\end{aligned}
$$

$$= \frac{n_2}{2}\Big\{ \sum_{v_i \in V(G_1)} \sum_{v_k \in V(G_1)} d_{G_1}(v_i) d_{G_1}(v_i, v_k) + 2n_1m_1 - PI(G_1)\Big\}$$
$$= \frac{n_2}{2}\Big\{ \sum_{\{v_i, v_k\} \subseteq V(G_1)} (d_{G_1}(v_i) + d_{G_1}(v_k)) d_{G_1}(v_i, v_k) + 2n_1m_1 - PI(G_1)\Big\}$$
$$= \frac{n_2}{2}\left\{DD(G_1) + 2n_1m_1 - PI(G_1)\right\},$$

$$W_7 := \sum_{v_i \in V(G_1)} \sum_{\substack{w_{ij} \in V_{i_v}(G_2) \\ v_k \in V(G_1)}} d_G(w_{ij}, v_k)$$
$$= n_2 \sum_{v_i \in V(G_1)} \Big\{ \sum_{v_k \in V(G_1)} d_{G_1}(v_i, v_k) + 2\Big\}$$
$$= n_2 \sum_{v_i \in V(G_1)} \sum_{v_k \in V(G_1)} d_{G_1}(v_i, v_k) + 2n_1n_2$$
$$= 2n_2(W(G_1) + n_1),$$

and

$$W_8 := \sum_{e_i = v_l v_m \in E(G_1)} \sum_{v_k \in V(G_1)} \sum_{\substack{u_{ij} \in V_{i_e}(G_2) \\ w_{km} \in V_{k_v}(G_2)}} d_G(u_{ij}, w_{km})$$
$$= (n_2^2/2) \sum_{e_i \in E(G_1)} \Big\{ \sum_{\substack{v_k \in V(G_1) \\ d(v_l, v_k) \neq d(v_k, v_m)}} (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1) +$$
$$\sum_{\substack{v_k \in V(G_1) \\ d(v_l, v_k) = d(v_k, v_m)}} (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 2) + 4\Big\}$$
$$= (n_2^2/2)(DD(G_1) + 2n_1m_1 - PI(G_1) + 4m_1) \quad \text{(similar to } W_6\text{)}.$$

Thus,

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u, v) = \sum_{i=1}^{8} W_i$$
$$= (n_2+1)^2 W(G_1) + n_2^2(W_e(G_1) - |E_\Delta(G_1)|) + \frac{n_2}{2}\Big\{(n_2+1)DD(G_1) -$$
$$(n_2+1)PI(G_1) + n_2(m_1^2 + (2n_1+9)m_1 + 2n_1) + 2((n_1-1)m_1 + n_1)\Big\} -$$
$$m_2(n_1 + m_1).$$

This completes the proof. □

**Theorem 3.2.** *The degree distance index of* $G_1 \circledast G_2$ *is given by*

$$\begin{aligned} DD(G) = {} & 4m_2(n_2+1)W(G_1) + n_2(3n_2+1)(Gut(G_1) + (C(G_1)/2)) + \\ & (2(n_2+1)^2 + (m_2+n_2)(2n_2+1) - 1)DD(G_1) + 4n_2(n_2+m_2)W_e(G_1) + \\ & 3n_2^2 M_1(G_1) - (m_1+n_1)M_1(G_2) - (n_2^2+n_2+m_2+2m_2n_2)PI(G_1) - \\ & n_2^2 T_1(G_1) - 4n_2m_2|E_\Delta(G_1)| + (5n_2^2 + (2m_2+1)n_2)m_1^2 + ((2n_1+18)n_2^2 + \\ & ((4n_1+18)m_2 + 2n_1)n_2 + 2m_2(n_1-6))m_1 + 4n_1m_2n_2. \end{aligned}$$

*Proof.* Using Lemmas 2.2 and 2.3, we get

$$\begin{aligned} D_1 := {} & \sum_{\{v_i,v_j\}\subseteq V(G_1)} (d_G(v_i) + d_G(v_j))d_G(v_i,v_j) \\ = {} & (2n_2+1) \sum_{\{v_i,v_j\}\subseteq V(G_1)} (d_{G_1}(v_i) + d_{G_1}(v_j))d_{G_1}(v_i,v_j) \\ = {} & (2n_2+1)DD(G_1), \end{aligned}$$

$$\begin{aligned} D_2 := {} & \sum_{e_i\in E(G_1)} \sum_{\{u_{ij},u_{ik}\}\subseteq V_{i_e}(G_2)} (d_G(u_{ij}) + d_G(u_{ik}))d_G(u_{ij},u_{ik}) \\ = {} & \sum_{e_i\in E(G_1)} \sum_{\{u_{ij},u_{ik}\}\subseteq V_{i_e}(G_2)} (d_{G_2}(u_j) + d_{G_2}(u_k) + 4)d_G(u_{ij},u_{ik}) \\ = {} & \sum_{e_i\in E(G_1)} \Big\{ 2 \sum_{\{u_j,u_k\}\subseteq V(G_2)} (d_{G_2}(u_j) + d_{G_2}(u_k) + 4) - \\ & \sum_{u_ju_k\in E(G_2)} (d_{G_2}(u_j) + d_{G_2}(u_k) + 4) \Big\} \\ = {} & \sum_{e_i\in E(G_1)} \Big\{ 2(n_2-1) \sum_{u_j\in V(G_2)} d_{G_2}(u_j) + 4(n_2-1)n_2 - M_1(G_2) - 4m_2 \Big\} \\ = {} & \sum_{e_i\in E(G_1)} \{4(n_2-1)(m_2+n_2) - M_1(G_2) - 4m_2\} \\ = {} & m_1(4(n_2-1)(m_2+n_2) - 4m_2 - M_1(G_2)), \end{aligned}$$

$$\begin{aligned} D_3 := {} & \sum_{v_i\in V(G_1)} \sum_{\{w_{ij},w_{ik}\}\subseteq V_{i_v}(G_2)} (d_G(w_{ij}) + d_G(w_{ik}))d_G(w_{ij},w_{ik}) \\ = {} & 4(n_2(n_2-1) - m_2)m_1 + 4n_1m_2(n_2-1) - n_1M_1(G_2) \text{ (similar to } D_2), \end{aligned}$$

$$\begin{aligned} D_4 := {} & \sum_{\{e_i,e_k\}\subseteq E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2) \\ u_{km}\in V_{k_e}(G_2)}} (d_G(u_{ij}) + d_G(u_{km}))d_G(u_{ij},u_{km}) \\ = {} & \sum_{\{e_i,e_k\}\subseteq E(G_1)} \Big\{ \sum_{\substack{u_{ij}\in V_{i_e}(G_2) \\ u_{km}\in V_{k_e}(G_2)}} (d_{G_2}(u_j) + d_{G_2}(u_m) + 4)(d_{G_1}(e_i,e_k) + 2) \Big\} \\ = {} & \sum_{\{e_i,e_k\}\subseteq E(G_1)} (4m_2n_2 + 4n_2^2)(d_{G_1}(e_i,e_k) + 2) \\ = {} & 4n_2(m_2+n_2)(W_e(G_1) + m_1(m_1-1)/2), \end{aligned}$$

$$
\begin{aligned}
D_5 := & \sum_{\{v_i,v_j\}\subseteq V(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v(G_2)}}} [d_G(w_{ik}) + d_G(w_{jm})]\ d_G(w_{ik}, w_{jm}) \\
= & \sum_{\{v_i,v_j\}\subseteq V(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i) + d_{G_1}(v_j) + d_{G_2}(u_k) + d_{G_2}(u_m))d_{G_1}(v_i, v_j) + \\
& 2\sum_{v_iv_j\in E(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i) + d_{G_1}(v_j) + d_{G_2}(u_k) + d_{G_2}(u_m)) - \\
& \sum_{v_iv_j\in E_\Delta(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i) + d_{G_1}(v_j) + d_{G_2}(u_k) + d_{G_2}(u_m)) \\
= & \sum_{\{v_i,v_j\}\subseteq V(G_1)} d_{G_1}(v_i,\ v_j)\ (n_2^2(d_{G_1}(v_i) + d_{G_1}(v_j)) + 4n_2m_2) + \\
& 2\sum_{v_iv_j\in E(G_1)} (n_2^2(d_{G_1}(v_i) + d_{G_1}(v_j)) + 4n_2m_2) - \\
& \sum_{v_iv_j\in E_\Delta(G_1)} (n_2^2(d_{G_1}(v_i) + d_{G_1}(v_j)) + 4n_2m_2) \\
= & n_2^2(DD(G_1) + 2M_1(G_1) - T_1(G_1)) + 4n_2m_2(W(G_1) + 2m_1 - |E_\Delta(G_1)|),
\end{aligned}
$$

$$
\begin{aligned}
D_6 := & \sum_{e_i=v_lv_m\in E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ v_k\in V(G_1)}} (d_G(u_{ij}) + d_G(v_k))d_G(u_{ij}, v_k) \\
= & \sum_{e_i=v_lv_m\in E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ v_k\in V(G_1)}} (d_{G_2}(u_j) + 2 + (2n_2+1)d_{G_1}(v_k))d_G(u_{ij}, v_k) \\
= & \sum_{e_i=v_lv_m\in E(G_1)} \sum_{v_k\in V(G_1)} (2m_2 + 2n_2 + n_2(2n_2+1)d_{G_1}(v_k))d_G(u_{i1}, v_k) \\
= & (1/2)\Big\{ \sum_{e_i=v_lv_m\in E(G_1)} \Big\{ \sum_{v_k\in V(G_1)} \Big\{(2m_2 + 2n_2 + n_2(2n_2+1)d_{G_1}(v_k))\times \\
& (d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1)\Big\} + \sum_{\substack{v_k\in V(G_1)\\ d(v_l,v_k)=d(v_k,v_m)}} (2m_2 + 2n_2 + n_2(2n_2+1)d_{G_1}(v_k))\Big\}\Big\} \\
= & (1/2)\Big\{ \sum_{e_i=v_lv_m\in E(G_1)} \sum_{v_k\in V(G_1)} 2(m_2+n_2)(d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1) + \\
& \sum_{e_i=v_lv_m\in E(G_1)} \sum_{v_k\in V(G_1)} n_2(2n_2+1)d_{G_1}(v_k)(d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m) + 1) + \\
& \sum_{e_i=v_lv_m\in E(G_1)} \sum_{\substack{v_k\in V(G_1)\\ d(v_l,v_k)=d(v_k,v_m)}} (2m_2 + 2n_2 + n_2(2n_2+1)d_{G_1}(v_k))\Big\} \\
= & (1/2)n_2(2n_2+1) \sum_{e_i=v_lv_m\in E(G_1)} \sum_{v_k\in V(G_1)} d_{G_1}(v_k)(d_{G_1}(v_l, v_k) + d_{G_1}(v_k, v_m)) + \\
& (m_2+n_2)(DD(G_1) - PI(G_1) + 2n_1m_1) + n_2(2n_2+1)((C(G_1)/2) + m_1^2)
\end{aligned}
$$

$$= (1/2)n_2(2n_2+1)\sum_{v_i\in V(G_1)}\sum_{v_iv_j\in E(G_1)}\sum_{v_k\in V(G_1)} d_{G_1}(v_k)d_{G_1}(v_i,v_k)+$$
$$(m_2+n_2)(DD(G_1)-PI(G_1)+2n_1m_1)+n_2(2n_2+1)((C(G_1)/2)+m_1^2)$$
$$= (1/2)n_2(2n_2+1)\sum_{v_i\in V(G_1)}\sum_{v_k\in V(G_1)} d_{G_1}(v_i)d_{G_1}(v_k)d_{G_1}(v_i,v_k)+$$
$$(m_2+n_2)(DD(G_1)-PI(G_1)+2n_1m_1)+n_2(2n_2+1)((C(G_1)/2)+m_1^2)$$
$$= (m_2+n_2)(DD(G_1)-PI(G_1)+2n_1m_1)+n_2(2n_2+1)\times$$
$$(Gut(G_1)+(C(G_1)/2)+m_1^2),$$

$$D_7 := \sum_{v_i\in V(G_1)}\sum_{\substack{w_{ij}\in V_{i_v}(G_2)\\ v_k\in V(G_1)}} (d_G(w_{ij})+d_G(v_k))d_G(w_{ij},v_k)$$
$$= \sum_{v_i\in V(G_1)}\sum_{\substack{w_{ij}\in V_{i_v}(G_2)\\ v_k\in V(G_1)}} (d_{G_2}(u_j)+d_{G_1}(v_i)+(2n_2+1)d_{G_1}(v_k))d_G(w_{ij},v_k)$$
$$= \sum_{v_i\in V(G_1)}\Big\{\sum_{v_k\in V(G_1)} (2m_2+n_2(d_{G_1}(v_i)+(2n_2+1)d_{G_1}(v_k)))d_{G_1}(v_i,v_k)+$$
$$4m_2+4n_2(n_2+1)d_{G_1}(v_i)\Big\}$$
$$= 2DD(G_1)(n_2^2+n_2)+4m_2(W(G_1)+n_1)+8n_2m_1(n_2+1),$$

$$D_8 := \sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)}\sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ w_{km}\in V_{k_v}(G_2)}} (d_G(u_{ij})+d_G(w_{km}))d_G(u_{ij},w_{km})$$
$$= \sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)}\sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ w_{km}\in V_{k_v}(G_2)}} (d_{G_2}(u_j)+d_{G_2}(u_m)+d_{G_1}(v_k)+2)d_G(u_{ij},w_{km})$$
$$= \sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)} (4m_2n_2+n_2^2(d_{G_1}(v_k)+2))\ d_G(u_{i1},w_{k1})$$
$$= (1/2)\Big\{\sum_{e_i=v_lv_m\in E(G_1)}\Big\{\sum_{v_k\in V(G_1)}\Big\{(4m_2n_2+n_2^2(d_{G_1}(v_k)+2))\times$$
$$(d_{G_1}(v_l,v_m)+d_{G_1}(v_k,v_m)+1)\Big\}+\sum_{\substack{v_k\in V(G_1)\\ d(v_l,v_k)=d(v_l,v_k)}} (4m_2n_2+n_2^2(d_{G_1}(v_k)+2))+$$
$$16m_2n_2+2n_2^2(d_{G_1}(v_l)+d_{G_1}(v_m)+4)\Big\}\Big\}$$
$$= (2m_2n_2+n_2^2)(DD(G_1)-PI(G_1)+2m_1(n_1+2))+n_2^2(Gut(G_1)+m_1^2+$$
$$(C(G_1)/2)+M_1(G_1)).$$

Thus we have

$$DD(G) = \sum_{\{x,y\}\subseteq V(G)} (d_G(x)+d_G(y))d_G(x,y)$$

$$
\begin{aligned}
&= \sum_{i=1}^{8} D_i \\
&= 4m_2(n_2+1)W(G_1) + (2(n_2+1)^2 + (m_2+n_2)(2n_2+1) - 1)DD(G_1) + \\
&\quad n_2(3n_2+1)(Gut(G_1) + (C(G_1)/2)) + 4n_2(n_2+m_2)W_e(G_1) - n_2^2T_1(G_1) + \\
&\quad 3n_2^2M_1(G_1) - (m_1+n_1)M_1(G_2) - PI(G_1)(n_2^2+n_2+m_2+2m_2n_2) - \\
&\quad 4n_2m_2|E_\Delta(G_1)| + (5n_2^2 + (2m_2+1)n_2)m_1^2 + ((2n_1+18)n_2^2 + \\
&\quad ((4n_1+18)m_2 + 2n_1)n_2 + 2m_2(n_1-6))m_1 + 4n_1m_2n_2.
\end{aligned}
$$

This completes the proof. □

**Theorem 3.3.** *The Gutman index of edge-neighborhood corona $G := G_1 \circledast G_2$ of $G_1$ and $G_2$ is given by*

$$
\begin{aligned}
Gut(G) = {} & 2m_2(m_2+4n_2+1)DD(G_1) + (3n_2+1)(2m_2+5n_2+1)Gut(G_1) + \\
& 4m_2^2W(G_1) + 4(n_2+m_2)^2W_e(G_1) - 2m_2n_2T_1(G_1) - n_2^2T_2(G_1) + \\
& (7n_2^2 + n_2 - m_2 + 6n_2m_2)M_1(G_1) + 2n_2^2M_2(G_1) - (n_1+5m_1)M_1(G_2) - \\
& (m_1+n_1)M_2(G_2) - 2m_2(m_2+n_2)PI(G_1) + (3n_2+1)(m_2+n_2)C(G_1) - \\
& 4m_2^2|E_\Delta(G_1)| + 2(n_2+m_2)(4n_2+m_2+1)m_1^2 + ((4n_1+18)m_2^2 + \\
& ((4n_1+36)n_2 - 12)m_2 + 2n_2^2 - 4n_2)m_1 + 4n_1m_2^2.
\end{aligned}
$$

*Proof.* From Lemmas 2.2 and 2.3, we have

$$
\begin{aligned}
G_1 :=& \sum_{\{v_i,v_j\}\subseteq V(G_1)} d_G(v_i)d_G(v_j)d_G(v_i,v_j) \\
&= (2n_2+1)^2 \sum_{\{v_i,v_j\}\subseteq V(G_1)} d_{G_1}(v_i)d_{G_1}(v_j)d_{G_1}(v_i,v_j) \\
&= (2n_2+1)^2 Gut(G_1),
\end{aligned}
$$

$$
\begin{aligned}
G_2 :=& \sum_{e_i\in E(G_1)} \sum_{\{u_{ij},u_{ik}\}\subseteq V_{i_e}(G_2)} d_G(u_{ij})d_G(u_{ik})d_G(u_{ij},u_{ik}) \\
&= \sum_{e_i\in E(G_1)} \sum_{\{u_{ij},u_{ik}\}\subseteq V_{i_e}(G_2)} (d_{G_2}(u_j)+2)(d_{G_2}(u_k)+2)d_G(u_{ij},u_{ik}) \\
&= \sum_{e_i\in E(G_1)} \Big\{2 \sum_{\{u_j,u_k\}\subseteq V(G_2)} (d_{G_2}(u_j)+2)(d_{G_2}(u_k)+2) - \\
&\quad \sum_{u_ju_k\in E(G_2)} (d_{G_2}(u_j)+2)(d_{G_2}(u_k)+2)\Big\} \\
&= m_1(4m_2^2 + 4(n_2-1)(2m_2+n_2) - 3M_1(G_2) - M_2(G_2) - 4m_2),
\end{aligned}
$$

$$
\begin{aligned}
G_3 :=& \sum_{v_i\in V(G_1)} \sum_{\{w_{ij},w_{ik}\}\subseteq V_{i_v}(G_2)} d_G(w_{ij})d_G(w_{ik})d_G(w_{ij},w_{ik}) \\
&= n_1(4m_2^2 - M_2(G_2)) - (n_1+2m_1)M_1(G_2) \\
&+ 8m_1m_2(n_2-1) + (n_2(n_2-1) - m_2)M_1(G_1) \text{ (similar to } G_2\text{)},
\end{aligned}
$$

$$
\begin{aligned}
G_4 &:= \sum_{\{e_i,e_k\}\in E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ u_{km}\in V_{k_e}(G_2)}} d_G(u_{ij})d_G(u_{km})d_G(u_{ij},u_{km})\\
&= \sum_{\{e_i,e_k\}\subseteq E(G_1)} \Big\{ \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ u_{km}\in V_{k_e}(G_2)}} ((d_{G_2}(u_j)+2)(d_{G_2}(u_m)+2))(d_{G_1}(e_i,e_k)+2)\Big\}\\
&= \sum_{\{e_i,e_k\}\subseteq E(G_1)} (4m_2^2+4n_2^2+8m_2n_2)(d_{G_1}(e_i,e_k)+2)\\
&= 4(m_2+n_2)^2(W_e(G_1)+m_1(m_1-1)/2),
\end{aligned}
$$

$$
\begin{aligned}
G_5 &:= \sum_{\{v_i,v_j\}\subseteq V(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} d_G(w_{ik})d_G(w_{jm})d_G(w_{ik},w_{jm})\\
&= \sum_{\{v_i,v_j\}\subseteq V(G_1)} d_{G_1}(v_i,v_k) \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i)+d_{G_2}(u_k))(d_{G_1}(v_j)+d_{G_2}(u_m))+\\
&\quad 2\sum_{v_iv_j\in E(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i)+d_{G_2}(u_k))(d_{G_1}(v_j)+d_{G_2}(u_m))-\\
&\quad \sum_{v_iv_j\in E_\Delta(G_1)} \sum_{\substack{w_{ik}\in V_{i_v}(G_2)\\ w_{jm}\in V_{j_v}(G_2)}} (d_{G_1}(v_i)+d_{G_2}(u_k))(d_{G_1}(v_j)+d_{G_2}(u_m))\\
&= \sum_{\{v_i,v_j\}\subseteq V(G_1)} d_{G_1}(v_i,v_k)\Big\{n_2^2d_{G_1}(v_i)d_{G_1}(v_j)+2n_2m_2(d_{G_1}(v_i)+d_{G_1}(v_j))+4m_2^2\Big\}+\\
&\quad 2\sum_{v_iv_j\in E(G_1)} \Big\{n_2^2d_{G_1}(v_i)d_{G_1}(v_j)+2n_2m_2(d_{G_1}(v_i)+d_{G_1}(v_j))+4m_2^2\Big\}-\\
&\quad \sum_{v_iv_j\in E_\Delta(G_1)} \Big\{n_2^2d_{G_1}(v_i)d_{G_1}(v_j)+2n_2m_2(d_{G_1}(v_i)+d_{G_1}(v_j))+4m_2^2\Big\}\\
&= n_2^2(Gut(G_1)+2M_2(G_1)-T_2(G_1))+2n_2m_2(DD(G_1)+2M_1(G_1)-T_1(G_1))+\\
&\quad 4m_2^2(W(G_1)+2m_1-|E_\Delta(G_1)|),
\end{aligned}
$$

$$
\begin{aligned}
G_6 &:= \sum_{e_i=v_lv_m\in E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ v_k\in V(G_1)}} d_G(u_{ij})d_G(v_k)d_G(u_{ij},v_k)\\
&= (2n_2+1)\sum_{e_i\in E(G_1)} \sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ v_k\in V(G_1)}} (d_{G_2}(u_j)+2)d_{G_1}(v_k)d_G(u_{ij},v_k)\\
&= 2(m_2+n_2)(2n_2+1)\sum_{e_i\in E(G_1)} \sum_{v_k\in V(G_1)} d_{G_1}(v_k)d_G(u_{i1},v_k)\\
&= (m_2+n_2)(2n_2+1)\Big\{\sum_{e_i\in E(G_1)} \Big\{\sum_{v_k\in V(G_1)} d_{G_1}(v_k)(d_{G_1}(v_l,v_k)+d_{G_1}(v_k,v_m)+1)+
\end{aligned}
$$

$$\sum_{\substack{v_k\in V(G_1)\\ d(v_l,\ v_k)=d(v_k,\ v_m)}} d_{G_1}(v_k)\Big\}\Big\}$$
$$= 2(m_2+n_2)(2n_2+1)(Gut(G_1)+(C(G_1)/2)+m_1^2),$$

$$\begin{aligned}
G_7 &:= \sum_{v_i\in V(G_1)} \sum_{\substack{w_{ij}\in V_{i_v}(G_2)\\ v_k\in V(G_1)}} d_G(w_{ij})d_G(v_k)d_G(w_{ij},v_k)\\
&= (2n_2+1)\sum_{v_i\in V(G_1)} \sum_{\substack{w_{ij}\in V_{i_v}(G_2)\\ v_k\in V(G_1)}} (d_{G_2}(u_j)+d_{G_1}(v_i))d_{G_1}(v_k)d_G(w_{ij},v_k)\\
&= (2n_2+1)\Big\{\sum_{v_i\in V(G_1)}\Big\{\sum_{v_k\in V(G_1)}(2m_2+n_2d_{G_1}(v_i))d_{G_1}(v_k)d_{G_1}(v_i,v_k)+\\
&\quad 2(2m_2+n_2d_{G_1}(v_i))d_{G_1}(v_i)\Big\}\Big\}\\
&= 2(2n_2+1)(m_2DD(G_1)+n_2Gut(G_1)+n_2M_1(G_1)+4m_1m_2),
\end{aligned}$$

$$\begin{aligned}
G_8 &:= \sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)}\sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ w_{km}\in V_{k_v}(G_2)}} d_G(u_{ij})d_G(w_{km})d_G(u_{ij},w_{km})\\
&= \sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)}\sum_{\substack{u_{ij}\in V_{i_e}(G_2)\\ w_{km}\in V_{k_v}(G_2)}} (d_{G_2}(u_j)+2)(d_{G_2}(u_m)+d_{G_1}(v_k))d_G(u_{ij},w_{km})\\
&= 2(m_2+n_2)\sum_{e_i=v_lv_m\in E(G_1)}\sum_{v_k\in V(G_1)}(2m_2+n_2d_{G_1}(v_k))d_G(u_{i1},w_{k1})\\
&= (m_2+n_2)\Big\{\sum_{e_i=v_lv_m\in E(G_1)}\Big\{\sum_{v_k\in V(G_1)}\Big\{(2m_2+n_2d_{G_1}(v_k))\times\\
&\quad (d_{G_1}(v_l,v_m)+d_{G_1}(v_k,v_m)+1)\Big\}+\sum_{\substack{v_k\in V(G_1)\\ d(v_l,v_k)=d(v_l,v_k)}}(2m_2+n_2d_{G_1}(v_k))+\\
&\quad 2n_2(d_{G_1}(v_l)+d_{G_1}(v_m))+8m_2\Big\}\Big\}\\
&= 2m_2(m_2+n_2)(DD(G_1)+2n_1m_1-PI(G_1)+4m_1)\\
&\quad +2n_2(m_2+n_2)(Gut(G_1)+m_1^2+\frac{C(G_1)}{2}+M_1(G_1)).
\end{aligned}$$

Thus we have

$$\begin{aligned}
Gut(G) &= \sum_{\{x,y\}\subseteq V(G)} d_G(x)d_G(y)d_G(x,y)\\
&= \sum_{i=1}^{8} G_i
\end{aligned}$$

$$= 2m_2(m_2 + 4n_2 + 1)DD(G_1) + (3n_2 + 1)(2m_2 + 5n_2 + 1)Gut(G_1)+ 4m_2^2W(G_1) + 4(n_2 + m_2)^2W_e(G_1) - 2m_2n_2T_1(G_1) - n_2^2T_2(G_1)+ (7n_2^2 + n_2 - m_2 + 6n_2m_2)M_1(G_1) + 2n_2^2M_2(G_1) - (n_1 + 5m_1)M_1(G_2)- (m_1 + n_1)M_2(G_2) - 2m_2PI(G_1)(m_2 + n_2) + (3n_2 + 1)(m_2 + n_2)C(G_1)- 4m_2^2|E_\Delta(G_1)| + 2(n_2 + m_2)(4n_2 + m_2 + 1)m_1^2 + ((4n_1 + 18)m_2^2+ ((4n_1 + 36)n_2 - 12)m_2 + 2n_2^2 - 4n_2)m_1 + 4n_1m_2^2.$$

This completes the proof. □

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